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II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; CHARLES C. CROSS, Libertytown, Md.; CHARLES E. MEYERS, Canton, Ohio, and J. SCHEFFER, A. M., Hagerstown, Md.

The equation of motion is $d^2x/dt^2 = -gx/R$. Where g =gravity=32½ feet, R =3963 miles=radius of earth. Multiplying by $2dx$ and integrating, we get $(dx/dt)^2 = -gx^2/R + C$.

Let $x=a$ when $dx/dt=0$. Then $C=ga^2/R$.

$$\therefore (dx/dt)^2 = (g/r)(a^2 - x^2) \dots (1).$$

$$\therefore dt = \sqrt{R/g} \cdot dx / \sqrt{a^2 - x^2}.$$

$$\therefore t = \sqrt{R/g} \cos^{-1}(x/a) + C_1.$$

Since $t=0$ when $x=a$, $C_1=0$.

$$\therefore t = \sqrt{R/g} \cos^{-1}(x/a) \dots (2).$$

Equation (2) represents simple harmonic motion. From (1), velocity is zero when $x=\pm a$ and a maximum when $x=0$.

When $x=0$, $t=\frac{1}{2}\pi\sqrt{R/g}$. This time is the same for any value of a between 0 and R .

$$2t = \text{time of complete vibration} = \pi\sqrt{R/g}.$$

$$2t = 42 \text{ minutes, 12 seconds.}$$

Also solved by J. H. DRUMMOND.

DIOPHANTINE ANALYSIS.

69. Proposed by JOSIAH H. DRUMMOND, LL. D., Counselor at Law, Portland, Me.

Two right angled triangles have the same base which is a mean proportional between the two perpendiculars: find a general solution, that will give integral values for all the sides of both triangles.

I. Solution by the PROPOSER.

Let x =the base, y =the perpendicular of one triangle, and z =that of the other triangle. Then $x^2=yz \dots (1)$, and $x^2+y^2=\square \dots (2)$, and $x^2+z^2=\square \dots (3)$. Substituting yz for x^2 in (2) and (3), we have $zy+y^2=\square \dots (4)$, and $zy+z^2=\square \dots (5)$. Take $y=mz$ and substituting in (4) and (5), and reducing, we have $m+1=\square$, say, p^2 ; and $m=p^2-1$.

$m(m+1)=\square \dots (6)$, substituting the value of m in (6), we have $p^2(p^2-1)=\square$. Hence we have to make $p^2-1=\square$, say, $(q-p)^2$. Hence $p=(q^2+1)/2q$ and $m=[(q^2-1)/2q]^2$; and $y=z[(q^2-1)/2q]^2$. $x=\sqrt{yz}=z[(q^2-1)/2q]$; to obtain integral values take $z=4q^2$, and we have $x=2q(q^2-1)$, $z=4q^2$ and hypotenuse $=2q(q^2+1)$, and $x=2q(q^2-1)$, $y=(q^2-1)^2$ and hypotenuse $=(q^2+1)(q^2-1)$, in which q may be any number >1 .

If the value of q is an odd number, it is manifest that one fourth of each of the foregoing values will be integral, and we shall have smaller values.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let p , h be the perpendiculars; c , d the hypotenuses; k the common base.

Then $p^2 + k^2 = c^2$, $h^2 + k^2 = d^2$, $ph = k^2$.

Let $p = a^2 + a^2b$, $h = ab^2 + b^3$.

$\therefore k^2 = a^2b^2(a+b)^2$.

$\therefore a^2(a+b)^2(a^2+b^2) = c^2$, $b^2(a+b)^2(a^2+b^2) = d^2$.

Let $a = m^2 - n^2$, $b = 2mn$, $m > n$.

$\therefore p = (m^2 - n^2)^2(m^2 - n^2 + 2mn)$,

$h = 4m^2n^2(m^2 - n^2 + 2mn)$,

$k = 2mn(m^2 - n^2)(m^2 - n^2 + 2mn)$,

$c = (m^2 - n^2)(m^2 - n^2 + 2mn)(m^2 + n^2)$,

$d = 2mn(m^2 - n^2 + 2mn)(m^2 + n^2)$.

Let $m = 2$, $n = 1$.

$\therefore p = 63$, $h = 112$, $k = 84$, $c = 105$, $d = 120$.

III. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.; SYLVESTER ROBINS, North Branch Depot, N. Y., and CHARLES C. CROSS, Libertytown, Md.

(a) Take $2mn$, $m^2 - n^2$, and $m^2 + n^2$, the general expressions for the sides of right triangles. Now multiply these sides, respectively, by $r(m^2 - n^2)$ and $2mnr$ and we obtain the respective sides of the two required triangles :

(1) $2mnr(m^2 - n^2)$, $r(m^2 - n^2)^2$, $r(m^2 - n^2)(m^2 + n^2)$,

(2) $2mnr(m^2 - n^2)$, $r(2mn)^2$, $2mnr(m^2 + n^2)$,

in which m , n and r may be any integers, $m > n$.

Take $n = r = 1$ and $m = 2$. Then the sides of the two triangles are, respectively, 12, 9, 15, and 12, 16, 20; for $12^2 = 9 \times 16$.

$m = 2$, $n = 1$, $r = 3$, gives 36, 27, 45, and 36, 48, 60.

$m = r = 3$, $n = 2$, gives 180, 75, 195, and 180, 432, 468.

[GRUBER, CROSS.]

(b) Multiply the sides of any integral right triangle by each leg, or the same number of times each leg, respectively, of the triangle, and the two results will be the respective sides of the two required triangles.

Take the sides 8, 15, 17. Multiply, respectively, by the legs 8 and 15, and we obtain the triangles 120, 64, 136, and 120, 225, 255.

Take the sides 5, 12, 13. Multiply, respectively, by 2×5 and 2×12 , and we obtain the triangles 120, 50, 130, and 120, 288, 312.

[ROBINS, GRUBER.]

IV. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio State University, Athens, Ohio.

Let $2mn$, $m^2 - n^2$, and $m^2 + n^2$; $2m'n'$, $m'^2 - n'^2$ and $m'^2 + n'^2$ be the pair of right triangles fulfilling the conditions

$$2mn = 2m'n' \dots (1),$$

$$\text{and } 4m^2n^2 = 4m'^2n'^2 = (m^2 - n^2)(m'^2 - n'^2) \dots (2).$$

$$\text{From (2), } m'^2 - n'^2 = \frac{4m^2n^2}{m^2 - n^2} \dots (3).$$

$$\text{Squaring (3), } m'^4 - 2m'^2n'^2 + n'^4 = \frac{16m^4n^4}{(m^2 - n^2)^2} \dots (4).$$

We have $4m'^2n'^2 = 4m^2n^2 \dots\dots (5)$.

Adding (4) and (5), $(m'^2 + n'^2)^2 = 4m^2n^2 \left(\frac{4m^2n^2}{(m^2 - n^2)^2} + 1 \right)$

$$\text{or } m'^2 + n'^2 = \frac{2mn(m^2 + n^2)}{m^2 - n^2} \dots\dots (6).$$

m and n must be selected so as to make (3) and (6) integral.

70. Proposed by CHARLES CARROLL CROSS, Libertytown, Md.

Give methods for decomposing numbers into squares, cubes, or biquadrates, and show that 61×200^3 is the sum of ten cube numbers, and that 844933 is the sum of eleven biquadrates in thirteen different ways. [From *The Mathematical Magazine*, Vol. II, No. 10.]

Comment by the PROPOSER.

Dr. Artemas Martin has written me that he knows of no method of separating a given number into squares, cubes and other powers, except *trial*.

I take the following numbers from *The Mathematical Magazine*.

$$61 \times 200^3 = 1^3 + 2^3 + 4^3 + 6^3 + 15^3 + 46^3 + 60^3 + 270^3 + 500^3 + 700^3.$$

$$\begin{aligned} 844933 = & 27^4 + 20^4 + 16^4 + 15^4 + 12^4 + 10^4 + 8^4 + 7^4 + 3^4 + 2^4 + 1^4, \\ & = 25^4 + 24^4 + 16^4 + 15^4 + 8^4 + 6^4 + 5^4 + 4^4 + 3^4 + 2^4 + 1^4, \\ & = 25^4 + 24^4 + 15^4 + 14^4 + 12^3 + 10^4 + 7^4 + 4^4 + 3^4 + 2^4 + 1^4, \\ & = 25^4 + 22^4 + 18^4 + 15^4 + 14^4 + 11^4 + 10^4 + 6^4 + 3^4 + 2^4 + 1^4, \\ & = 25^4 + 21^4 + 18^4 + 17^4 + 14^4 + 12^4 + 10^4 + 6^4 + 5^4 + 4^4 + 1^4, \\ & = 25^4 + 21^4 + 18^4 + 16^4 + 15^4 + 12^4 + 10^4 + 9^4 + 6^4 + 3^4 + 2^4, \\ & = 25^4 + 21^4 + 18^4 + 15^4 + 14^4 + 13^4 + 12^4 + 10^4 + 8^4 + 7^4 + 2^4, \\ & = 25^4 + 20^4 + 19^4 + 18^4 + 14^4 + 10^4 + 9^4 + 7^4 + 6^4 + 4^4 + 3^4, \\ & = 25^4 + 20^4 + 19^4 + 16^4 + 14^4 + 13^4 + 11^4 + 10^4 + 9^4 + 4^4 + 2^4, \\ & = 24^4 + 23^4 + 18^4 + 15^4 + 14^4 + 12^4 + 10^4 + 9^4 + 6^4 + 5^4 + 3^4, \\ & = 24^4 + 21^4 + 20^4 + 16^4 + 15^4 + 13^4 + 10^4 + 7^4 + 6^4 + 4^4 + 1^4, \\ & = 24^4 + 21^4 + 20^4 + 16^4 + 15^4 + 12^4 + 11^4 + 8^4 + 7^4 + 5^4 + 2^4, \\ & = 24^4 + 21^4 + 19^4 + 18^4 + 15^4 + 12^4 + 10^4 + 6^4 + 5^4 + 3^4 + 2^4. \end{aligned}$$

MISCELLANEOUS.

64. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

How many bushels of wheat will a conical bin 8 feet in diameter at base and 12 feet high hold, if part of the bin is cut off by a plane parallel to the side and passing through the center of the base?

Solution by the PROPOSER.

Let ABC be the cone, DF the plane.

Let $BD=c$, $\tan DBC = \cot FDC = n$, $DC=R$.

$\therefore x^2 + z^2 = n^2(c-y)^2$ is the equation to the cone.

$x=ny$ is the equation to the plane.

V =volume of cone cut off by plane.